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## Archimedes' Principle

## Purpose

To connect the words of Archimedes' Principle to the actual behavior of submerged objects.
To examine the cause of buoyancy, that is the variation of pressure with depth in a fluid.
To use Archimedes’ Principle to determine the density of an unknown material.

## Explore the Apparatus

## Virtual Buoyancy Apparatus

## PENCIL

See Video Overview at: http://virtuallabs.ket.org/physics/apparatus/11_buoyancy/
We'll use the Buoyancy Apparatus in this lab activity. You can get quick access to help by rolling your mouse over most objects on the screen.


Figure 1 - The Buoyancy Apparatus
To investigate buoyant forces we need to measure the weight and volume of objects as well as their submerged weight when fully or partially immersed in a fluid. We'll use water as our fluid in this lab. We also need to measure the weight and volume of the fluid displaced. A hanging scale and a digital scale are available for measuring the weights of our objects. The overflow of water from the tank spills into a graduated cylinder. The volume of this displaced water can be read from the graduated cylinder; and the weight of the water can be found by weighing the graduated cylinder before and after overflow.

Three simple and three compound objects are available for study. They are composed of cork, aluminum, wood, and two unknown materials. These objects can be weighed in air and when partially or fully submerged. The hanging scale is raised
and lowered by clicking and dragging it. Both the hanging scale and the graduated cylinder can be read more precisely by zooming in. The graduated cylinder is emptied by dragging and releasing it above the tank.

You'll find that some actions that could be done with real apparatus are forbidden. For example, you can't weigh more than one object on the digital scale at a time. And you can't make a mess by lowering an object into the water while the graduated cylinder is on the digital scale. Another thing that is largely forbidden is dragging the screen when you're zoomed in. When you try to move an on-screen object and it doesn't behave as expected this means that something else needs to be done first. There aren't too many choices.

## Archimedes' Principle:

## A body wholly or partially immersed in a fluid will experience a buoyant force equal to the weight of the fluid displaced.

This and many other phrases you'll hear when studying buoyancy are mostly meaningless without direct experience. So our exploration of the apparatus will mainly focus on the various situations you'll encounter and the related terminology. While we're at it, our use of submerged in some cases and immersed in others is not meant to suggest two different phenomena. Both refer to an object fully or partially in a fluid.

The following exploration, down to the Theory section, is not a part of the graded lab. Instead it's meant to clarify some concepts and terms that make better sense if you get some practice with them.

## 1. What's a "buoyant force" and what exerts it?

Let's try it with the system in Figure 1. Right (or Ctrl) click and zoom in on the hanging scale. It appears to read zero initially. This sort of scale is not very precise. Our simulation reflects that level of precision. (Zoom back out.)

Now drag the cork and aluminum compound object and drop it somewhere below the hanger. It should become attached to the hanger. You should find that it weighs about 4.4 N . Zooming in and estimating a digit we might have 4.41 N . Drag and drop it on the digital scale and you'll get 4.41 N . Which scale should you use? We'll always use the value from the hanging scale, 4.41 N in this case, unless we have to use the digital scale for a measurement. This will only happen when we need to weigh the graduated cylinder.

Now for the "(submerged) weight when fully or partially immersed in the water." Drag the object back to the hanging scale. Slowly drag the scale downward until the aluminum disk is about half submerged. The scale might read about 3.73 N. This is a judgment call since the location of the half-way point is uncertain. Let's use that value in the following discussion. That's the "submerged weight." We'll use the term $W^{\prime}$ (" $W$ prime") to represent the submerged weight.

So its actual weight is 4.41 N and it appears lighter when partially submerged, maybe 3.73 N . It didn't lose weight. The water is providing an upward force on the bottom of the disk. So we would say that a $4.41-\mathrm{N}$ object was buoyed up by a force $F_{\mathrm{b}}$ so that it appears to weigh, or has a submerged weight of, 3.73 N . Since it's in vertical equilibrium, we can say $\Sigma \mathbf{F}_{\mathrm{y}}=0$.

Let's look at the forces and draw vector arrows to represent each force.

We know the object has some actual weight, a downward force, $\mathbf{W}$. We measure this force with our scale by finding the upward force that balances it. But the weight, $\mathbf{W}$, is a downward force.
When the object is somewhat submerged the scale exerts a lesser upward force, $\mathbf{W}^{\prime}$.
Since the pair of disks is still in equilibrium, some additional upward force must make up the difference. This is the buoyant force, $\mathbf{F}_{\mathbf{b}}$.

Since the object is still in equilibrium the three vector arrows shown in Figure 2a must vectorially add to equal zero. This is shown in two ways in Figures 2b and 2c. Figure 2c is a free-body diagram, FBD, where vectors radiate away from a point representing the object that the forces act on. We'll make use of FBDs throughout this lab to help us picture the forces acting on an object and to help us create equations to relate the force involved.


Our FBD guides us to add the three force vectors together to equal a net force of zero.

$$
\begin{aligned}
& \Sigma \mathbf{F}_{y}=\mathbf{W e i g h t}_{\text {submerged }}+\mathbf{F}_{\mathrm{b}}+\mathbf{W} \text { eight }=0 \\
& \Sigma \mathbf{F}_{y}=\mathbf{W}^{\prime}+\mathbf{F}_{\mathrm{b}}+\mathbf{W}=0
\end{aligned}
$$

Figure 2

In scalar form, which we'll use from here on, we add the magnitudes of the forces together with signs indicating their directions. Since only the weight is downward,

$$
W^{\prime}+F_{\mathrm{b}}-W=0
$$

so

$$
F_{\mathrm{b}}=W-W^{\prime}
$$

Getting back to our measured values,
a. $\quad F_{\mathrm{b}}=W-W^{\prime}=4.41 \mathrm{~N}-3.73 \mathrm{~N}=$ $\qquad$ N

This value, 0.68 N , is the buoyant force? The buoyant force is the part of the weight that the scale no longer has to support because a new upward force is present. It's an upward force caused by the pressure on the bottom of the disk. If you lower the disk further, the buoyant force becomes larger and the submerged weight, $W^{\prime}$, is reduced by an equal amount. Try it.

## 2. What is the "fluid displaced" and how is its weight related to the buoyant force?

In the preceding section you focused on the buoyant force as an apparent reduction in the weight of the submerged object. What's the source of this force?

With the cork and aluminum disk pair attached to the scale, lower the scale as far as it will go. A lot of water will overflow into the cylinder. When that's done, raise the scale back up. Now gradually lower the disks into the water. Notice the behavior of the water. Its level rises because the disks are "displacing" some of the water. That is, they're taking the water's place. So it rises to make room.

When the water occupied that space, it was supported by the water below it. When the disks occupy the same space, the water below them provides exactly the same amount of upward force. That's the buoyant force. It was there all the time, but it was just holding up the water until the disks arrived.

So "the buoyant force is equal to the weight of the fluid displaced!" We need to find out how to determine that weight.
Refill the tank. To do this just drag the graduated cylinder somewhere above the tank and release it. It will empty itself and return to its post. Now gradually lower the disks into the water and notice what happens to the "displaced water." Since our tank was initially full this time, the displaced water overflowed into the graduated cylinder. We did that so that we could weigh it or find its volume. Let's use our previous results to see the relationship between the buoyant force and the displaced fluid's weight. If your apparatus is not right where you left it at the end of Part 1, repeat Part \#1 to restore it. Be sure to zoom in once for better precision.

Insert your value from 1a above in the following blank.
b. The displaced water's weight = the buoyant force on the disk = $\qquad$ N
c. Drag the graduated cylinder onto the digital scale. Scale reading with cylinder and water $=$ $\qquad$ N
That's the weight of the fluid plus the weight of the graduated cylinder. So we need to find the weight of the graduated cylinder and subtract it from the total to find the weight of the water. Empty the cylinder as before. Place the cylinder on the digital scale.
d. Weight of empty graduated cylinder, $W_{\mathrm{c}}=$ $\qquad$ N
e. $\quad F_{\mathrm{b}}=$ weight of water displaced $=W_{\text {water }+ \text { cylinder }}-W_{\text {cylinder }}=$ $\qquad$ N
(Ex. $1.69-.98) \mathrm{N}=.71 \mathrm{~N})$
Got it? We'll see why your values for (b) and (e) were about the same shortly. But for now you should understand what we mean by the statement that the buoyant force equals the weight of the fluid displaced.

## 3. Finding and using the volume of the fluid displaced.

Now suppose we didn't have either of our scales. Could you still determine $F_{b}$ ? We know that the weight of the water can be found from $W_{\text {water }}=m_{\text {water }} g=\rho_{\text {water }} g V_{\text {water }}$. Since we know that the density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$, we could calculate the weight of the water if we knew its volume. The first thing you need to do is lower the masses back into the water to about where they were before.
You can zoom in on the cylinder to read the volume reading but there's a "Cylinder Zoom" feature that does that for you. You can now read the volume in ml .

## NOTE:

The graduated cylinder measures volume in ml . You'll always need to convert it to $\mathrm{m}^{3}$. ( $1 \mathrm{ml}=1 \mathrm{~cm}^{3}=10^{-6} \mathrm{~m}^{3}$.)
f. $\quad \mathrm{Fb}=W_{\text {water displaced }}=\rho_{\text {water }} g V_{\text {water }}=1000 \mathrm{~kg} / \mathrm{m}^{3} \times 9.80 \mathrm{~N} / \mathrm{kg} \times$ $\qquad$ $m^{3}=$ $\qquad$ N
(Ex. $1000 \mathrm{~kg} / \mathrm{m}^{3} \times 9.80 \mathrm{~N} / \mathrm{kg} \times 72 \times 10^{-6} \mathrm{~m}^{3}=.71 \mathrm{~N}$ )
How about the aluminum disk? If you weighed it and then completely submerged it you could measure its volume and then compute its density and convince the king that his disk is not silver.
Incidentally, we went to extra expense in providing a non-wetting" polypropylene graduated cylinder in this apparatus to avoid the need to account for a meniscus. You're welcome.

## Theory

## Archimedes' Principle

Archimedes' Principle is very easy to state in words, but learning how to use it takes practice in a variety of situations. In this lab we'll first explore the literal meaning of this principle. Then we'll put it to use in a couple of situations. You'll use the wording of the principle to draw free-body diagrams (FBDs) and create equations that can be used to find unknown quantities such as the volume, mass, or density of an object or of the fluid it's immersed in.
Solving problems using Archimedes' Principle can be really puzzling. The most important step is to use the words of the principle to create an equation. One important tool to help you create your equation is a free-body diagram. Archimedes’ Principle usually involves objects in vertical equilibrium. And the terms that go to into $\Sigma \mathbf{F}_{\mathrm{y}}=0$ will likely come from the words of Archimedes' Principle - a body (mass, weight) buoyed up by (force), etc. In case you've forgotten...

## Archimedes' Principle:

## A body wholly or partially immersed in a fluid will experience a buoyant force equal to the weight of the fluid displaced.

Without a bit more information, this is little more than magical incantation! We need to dig a little deeper and learn what causes the buoyant force and why the force varies as objects are gradually immersed in a fluid. We'll start with a couple of interactive animations built into the lab apparatus.

## Variation of Pressure with Depth

Click the Pressure button to load an interactive simulation. Adjust the small cylinder up and down as needed to recreate the discussion that follows.

In Figure 3a a solid brass cylinder is shown above a tank completely filled with water. Atmospheric pressure acts essentially equally on all sides of the cylinder and on the water's surface as indicated by the five $\mathbf{P}_{\text {atm }}$ arrows in Figure 3a. At all times the horizontal pressures will cancel. So we need only examine the vertical pressures.

Gradually lower the cylinder until about $1 / 4$ of it is submerged as shown in Figure 3b. (The water overflowing the sides is not shown. It also magically returns as needed.) Notice the vectors that appear and change as you move the cylinder up and down. A black vector, $\mathbf{P}_{\mathrm{up}}$, the upward pressure exerted by the water, is shown in two places along with a blue $\Delta \mathbf{P}$ vector.

1. What happens to the upward pressure, $P_{\mathrm{up}}$, and $\Delta P$ as the cylinder is lowered? Don't let the cylinder become completely submerged for now.


3a


Figure 3

At a given depth, $h$, in a container of water exposed to the atmosphere there is a pressure due to the weight of the column of water above that point plus the weight of the atmosphere above the surface. This pressure is omnidirectional and given by

$$
\begin{equation*}
P=\rho_{\mathrm{w}} g h+P_{\mathrm{atm}} \tag{1}
\end{equation*}
$$

We're only interested in the pressures at the top, $P_{\text {down }}$ and bottom, $P_{\text {up }}$ of the cylinder. We'll refer to these more generally as $P_{1}$ and $P_{2}$ respectively. With the cylinder partially submerged, only the upward pressure on the bottom of the cylinder appears. (See Figure 4a.)

What about $\Delta P$ ? $\Delta P$ is the difference in pressure between the top and bottom of the cylinder. Currently the top of the cylinder has a pressure $P_{\text {down }}=P_{\text {atm }}$ acting on it. The pressure acting on the bottom is $\mathrm{P}_{\text {up }}$ which equals $\mathrm{P}_{\mathrm{atm}}+\rho_{\mathrm{w}} g h_{2} . \Delta P$, the difference in these two pressures is just $P_{\text {up }}-P_{\text {down }}=\rho_{\mathrm{w}} g h_{2}$.

This explains why $\Delta P$ always (so far) equals $P_{\text {up }}$. Let's change that.

As you continue to drag the cylinder downward, there will continue to be just the three vectors and each will continue to increase in length but remain equal to each other until a specific point is reached.
2. Where is the top of the cylinder when this transition occurs?

Continue to lower the cylinder until it's about halfway to the bottom of the tank as shown in Figure 4 b . You can now clearly see the downward vector $\mathbf{P}_{\text {down }}$.

Drag the cylinder up and down in this middle region of the tank. As you do this the vector $\Delta \mathbf{P}$ behaves differently than the way it behaved before.


4a


4b

Figure 4
3. What happens to $\Delta P$, as the cylinder is lowered while completely submerged?
4. What happens to $P_{\text {up }}$, and $P_{\text {down }}$ as the cylinder is lowered while completely submerged?

How does this account for your answer to the previous question.

In summary, the difference in pressure from top to bottom is just $P_{2}-P_{1}=\rho g\left(h_{2}-h_{1}\right)$. In the case of a partially submerged object, $h_{1}=0$, so this equation will work for either a partially or fully submerged object.

## The Effect of the Pressure Difference, $\Delta P$, on the Buoyant Force

We've found a simple equation that gives the pressure difference between the top and bottom of our cylinder when it's somewhat or totally submerged. So how does this relate to buoyancy?


5a


5b

Figure 5: Note that $5 \mathbf{a}$ is the same as $\mathbf{3 b}$, and $5 b$ is the same as $\mathbf{4 b}$.
For a cylinder of top and bottom surface area, $A_{c}$, the total force exerted by the water pressure would be

$$
F_{\text {net water }}=F_{2}-F_{1}=\left(P_{2}-P_{1}\right) A_{\mathrm{c}}=\rho g\left(h_{2}-h_{1}\right) A_{c}
$$

Consider a cylinder of height $h_{\mathrm{c}}$.
In Figure 5a, with $h_{2}=\frac{1}{4} h_{\mathrm{c}}, \rho_{\mathrm{w}} g\left(h_{2}-h_{1}\right)$ reduces to $\rho_{\mathrm{w}} g\left(\frac{1}{4} h_{\mathrm{c}}\right)$
The total force, $F_{\text {net water }}$, exerted by the water pressure on the cylinder would be

$$
\begin{aligned}
& F_{\text {net water }}=\rho_{\mathrm{w}} g\left(\frac{1}{4} h_{\mathrm{c}}\right) A_{c} \\
&=\frac{1}{4} \rho_{\mathrm{w}} g V_{\mathrm{c}} \\
&=\text { the weight of the water that would occupy } 1 / 4 \text { the volume of the cylinder } \\
& \boldsymbol{F}_{2}=\text { "the weight of the water displaced" }
\end{aligned}
$$

Now consider the totally submerged cylinder in Figure 5b.
In this case, $h_{2}-h_{1}=h_{\mathrm{c}}$, so $\rho_{\mathrm{w}} g\left(h_{2}-h_{1}\right)$ reduces to $\rho_{\mathrm{w}} g h_{\mathrm{c}}$
The total force, $F_{\text {net water }}$, exerted by the water pressure on the cylinder would be

$$
\begin{aligned}
& F_{\text {net water }}=\rho_{\mathrm{w}} g h_{\mathrm{c}} A_{c} \\
& \\
& =\rho_{\mathrm{w}} g V_{\mathrm{c}} \\
& \\
& =\text { the weight of the water that would occupy the total volume of the cylinder } \\
& \boldsymbol{F}_{2}
\end{aligned}=\text { "the weight of the water displaced" }
$$

So no matter what volume of the cylinder is submerged, the buoyant force acting on it equals the weight of an equal volume of water.

What about different shapes? How about a cone or a sphere? The result is exactly the same but the proof is not so simple. We won't pursue that any farther.

Consider three cases: a cylinder of aluminum, a cylinder of liquid water, and cylinder of cork. A cylinder of liquid water would be impossible to create but we can imagine it to be enclosed in a thin, weightless, cylindrical shell.
5. As a cylinder of aluminum is lowered into our tank, when will the buoyant force, $F_{\mathrm{b}}$, exceed the cylinder's weight?
(a) when it's partially submerged;
(b) when it's fully submerged;
(c) never
6. Why is this the case?
7. Once the cylinder of aluminum is fully submerged we might hold it in place by attaching which of the following?
(a) a block of cork;
(b) a block of aluminum;
(c) a block of water;
(d) None necessary

Click the Buoyancy button. Select the aluminum cylinder. Use this to test your reasoning and correct it if necessary.
8. As a cylinder of water is lowered into our tank, when will the buoyant force, $F_{b}$, exceed the cylinder's weight?
(a) when it's partially submerged;
(b) when it's fully submerged;
(c) never
9. Why is this the case?
10. Once the cylinder of water is fully submerged we might hold it in place by attaching which of the following?
(a) a block of cork;
(b) a block of aluminum;
(c) a block of water;
(d) none necessary

Select the water cylinder. Use this to test your reasoning and correct it if necessary.
11. As a cylinder of cork is lowered into our tank, when will the buoyant force, $F_{b}$, exceed the cylinder's weight?
(a) when it's partially submerged;
(b) when it's fully submerged;
(c) never
12. Why is this the case?
13. Once the cylinder of cork is fully submerged we might hold it in place by attaching which of the following?
(a) a block of cork;
(b) a block of aluminum;
(c) a block of water;
(d) one necessary

Select the cork cylinder. Use this to test your reasoning and correct it if necessary.
14. It's useful to leave all three cylinders fully submerged at about the same depth and then click the different icons to switch among the three materials. There is one thing that is the same regardless of the cylinder chosen. What is it? Why?

One final suggestion. Reopen the Pressure simulation. Lower the cylinder all the way to the bottom. Any thoughts?

## Archimedes' Principle with Additional Applied Forces.

Most of our work with Archimedes’ principle includes additional forces such as the ones suggested at the end of each of the three sets of questions above. Typical examples include

- A lifeguard pulling a person up onto a dock will find that the person seems to get heavier as she’s lifted further out of the water. This leads to the strategy of starting out with a large upward pull to give the swimmer an initial upward speed, making the rest of the pull easier.
- A buoy in a river will be held in place by a heavy weight sitting on the river bottom. The "one final question" above will come into play in this case.
- The buoyant force might be used to partially or fully support another object. For example a life jacket has to supply enough buoyant force to support its own weight as well as the person wearing it. A water-logged life-jacket might continue to supply the same lift, but the added water will provide an additional undesirable downward force.

Analyzing such situations usually involves the use of translational equilibrium equations. That is, if a body is in equilibrium, the sum of the forces in any direction will equal zero. A buoyant force is just another force. It's just a bit trickier because it varies as an object is submerged to varying depths.

Just as important, a careful free-body diagram, FBD, is essential in working out what objects and forces are involved.
Example 1. A steel anchor weighs 2000 N in air. How much force would be required to support it when fully submerged in water? That is, what is its apparent weight, $\mathrm{W}^{\prime}$, when submerged? $W^{\prime}$ is the unknown upward support force in this case.
In addition to the upward support force, $W^{\prime}$, there is the upward buoyant force, $F_{\mathrm{b}}$, and the downward gravitational force, that is, its weight, $W$.
The sum of these forces equals zero. So, (using magnitudes) $F_{\mathrm{b}}+W^{\prime}=\mathrm{W}$
Other considerations:
$F_{\mathrm{b}}=$ weight of fluid (water) displaced
$=\rho_{\text {water }} g V_{\text {water }}=\rho_{\text {water }} g V_{\text {anchor }}$ since the anchor displaces an equal volume of water.
And since we know the weight and mass of the anchor we can find its volume by looking up its density and using $\rho_{\text {steel }}=m_{\text {anchor }} V_{\text {anchor }}$
What will change as the anchor breaks the surface of the water?


Figure 6

## Procedure

In the following sections you'll collect and analyze data to investigate Archimedes’ Principle in some typical situations such as the one suggested in Example 1. You won't be provided step by step directions but you'll have blanks provided for the data you need to collect. This method is being used because buoyancy problems are always sort of free form. It's not about finding an equation that fits. Rather you try to find a way that your information fits into Archimedes’ Principle.

You'll also get guidance in developing FBDs as well as equations describing your FBDs. Note that the equations are in scalar form. That is, we'll say
"The sum of the upward forces = the sum of the downward forces" (all positive magnitudes) rather than "The sum of the upward forces + the sum of the downward forces $=0 . "$ (All vectors).

## I. Confirm Archimedes' Principle for the case where $\rho_{\text {object }}>\rho_{\text {liquid }}$ using overflow

In this section we'll directly measure the buoyant force and the weight of the water displaced for an object that sinks. We'll then confirm that they are equal. Our object will be an aluminum disk.

If either the Buoyancy or Pressure simulation is still open, click the Close button.


7a


7b


7c


7d

Figure 7
Direct measurement of the buoyant force: Weighing the aluminum disk in air (Fig. 7a) gives us W. Weighing it when submerged (Fig. 7c) gives us $W^{\prime}$. From our FBD (7e) we can see that

$$
F_{\mathrm{b}}+W_{\mathrm{al}}^{\prime}=W_{\mathrm{al}},
$$

So

$$
\begin{equation*}
F_{\mathrm{b}}=W_{\mathrm{al}}-W_{\mathrm{al}}^{\prime} \tag{2}
\end{equation*}
$$

Measurement of the weight of the water displaced: Weighing the empty graduated cylinder (7b) gives us $W_{\text {cyl }}$. Weighing the cylinder after receiving the overflow water (7d) gives us $W_{\text {cyl + water displaced }}$.


FBD 7e

$$
\begin{equation*}
F_{\mathrm{b}}=W_{\text {water displaced }}=\left(W_{\mathrm{cyl}}+\text { water displaced }\right)-W_{\mathrm{cyl}} \tag{3}
\end{equation*}
$$

1. Collect and record the data below.
$W_{\mathrm{al}}=$ $\qquad$ N
$W_{\mathrm{cyl}}=$ $\qquad$ N
$W_{\mathrm{al}}^{\prime}=$ $\qquad$ N
$W_{\text {cyl }}$ + water $=$ $\qquad$ N
2. Calculate the buoyant force, $F_{\mathrm{b}}$. twice, using Equation 2, and Equation 3.
$F_{\mathrm{b}}=$ $\qquad$ N
(Equation 2)
$F_{\mathrm{b}}=$ $\qquad$ N (Equation 3)
$\%$ difference $=$ $\qquad$ \%

Show your calculations of $\boldsymbol{F}_{\mathrm{b}}$ for each method and the percentage difference between the two.

## II. Confirm Archimedes' Principle for pobject < $\rho l i q u i d ~ u s i n g ~ o v e r f l o w ~$

In this section we'll directly measure the buoyant force and the weight of the water displaced for a floating object. And we'll confirm that they are again equal. Our object will be a cork disk. This time you fill in all the missing pieces. Use part I as a model. Collect and record the data required below.

Also fill in the missing pieces below including Figures 8a, c, and d, FBD 8e

1. In the questions that follow you'll submit your responses for Figure 8a, 8c, and 8d by supplying the missing pieces scale pointer or digital scale reading. Either draw in the missing pieces or paste copies of Screenshots over the partial images below. For screenshots of 8a and 8c just capture an image of the scale. For 8d, capture an image of the digital scale reading.

2. Weighing the cork disk in air (Fig. 8a) gives us $W_{\text {cork. }}$. Weighing it when partially submerged (Fig. 8c) gives us $W^{\prime}$ cork. This time it floats. (Note the slack string.) So
$W_{\text {cork }}^{\prime}=$ $\qquad$ N
3. Draw FBD 8e to the left of the floating cork provided. There should be two force vectors. Optionally, use the Sketch tool $\curvearrowleft$ to create a screen-sketch by adding and labeling a pair of vector arrows to the left of the floating cork in the Buoyancy Apparatus. Take a Screenshot of the FBD and the cork and upload it as "Buoy_FBD8e.png". Paste the sketch over FBD 8e.

Use Equations 2 and 3 as a guide in producing Equations 4 and 5.
4. From FBD 8e we can say that
$F_{\mathrm{b}}=$ $\qquad$
Weighing the empty graduated cylinder (8b) gives us $W_{\text {cyl }}$. Weighing the cylinder after receiving the

## F68

$F_{\mathrm{b}}=$ $\qquad$
5. $W_{\text {cork }}=$ $\qquad$ N
6. $W_{\mathrm{cyl}}=$ $\qquad$ N
7. $W_{\text {cork }}^{\prime}=$ $\qquad$ N
8. $W_{\mathrm{cyl}}+$ water $=$ $\qquad$ N

Calculate the buoyant force, $F_{\mathrm{b}}$. twice, using Equation 4, and Equation 5.
9. $F_{\mathrm{b}}=$ $\qquad$ N (Equation 4)
10. $F_{\mathrm{b}}=$ $\qquad$ N
(Equation 5)
11. $\%$ difference $=$ $\qquad$ \%

Show your calculations of $\boldsymbol{F}_{\mathrm{b}}$ for each method and the percentage difference.

## III. Determine the density of an unknown material.

We've confirmed that Archimedes' Principle correctly describes the forces acting on a body wholly or partially submerged. That gives us a starting point for more complex situations that are not so straight-forward. One of your object choices is a compound object with a cork on top and a rod of negligible volume connecting it to a large, black unknown object below it. It's in the right-hand column. The unknown object is more dense than cork; otherwise the system would try to flip upsidedown. Your task is to determine the density of the unknown object. In the process you'll first have to determine the density of the cork.

## Density of the cork

You already know the weight of the cork from part II. From the weight you can calculate the mass. What you need is the cork's volume. You can find that using another compound object-the cork with the attached aluminum disk. It's in the middle of the left column of objects.

1. Attach the cork/aluminum object to the hanging scale. Lower the system slowly while watching the water overflow into the cylinder. Continue until the aluminum cylinder becomes completely submerged. Use "Cylinder Zoom" to read and record the volume of water displaced by the aluminum.

Continue a bit further to see that the connecting rod is ignored, causing no overflow. Continue on and notice that the cork becomes completely submerged. You want to know the volume of the cork. Fill in the data table below to find the volume of the cork, convert it to $\mathrm{m}^{3}$, and compute its density in $\mathrm{kg} / \mathrm{m}^{3}$.

1. $V_{\mathrm{al}}=$ $\qquad$ ml

## Show the calculations indicated.

2. $V_{\text {all }+ \text { cork }}=$ $\qquad$ ml

$$
V_{\mathrm{c}}(\mathrm{ml})=-
$$

3. $V_{\text {cork }}=$ $\qquad$ ml
4. $V_{\text {cork }}=$ $\qquad$ $\mathrm{m}^{3}$

$$
V_{\mathrm{c}}\left(\mathrm{~m}^{3}\right)=
$$

5. $W_{\text {cork }}=$ $\qquad$ N (from part II)

$$
\rho_{\mathrm{c}}=
$$

6. $m_{\text {cork }}=$ $\qquad$ kg
7. $\rho_{\text {cork }}=$ $\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$

## Finding the Density of an Unknown Material

Return the cork/aluminum object to its home. Empty the graduated cylinder.
8. Attach unknown system \#1 (bottom right) to the hanging scale. Gradually lower it as far down as it will go. What is different about this compound system relative to the cork/aluminum system?
9. Draw FBD 9 to the left of the floating cork-unknown system provided. Optionally, use the Sketch tool $\curvearrowleft$ to create a screen-sketch by adding and labeling vector arrows to the left of the floating cork in the Buoyancy Apparatus. Take a Screenshot of the FBD and the cork and upload it as "Buoy_FBD9.png". Paste the sketch over FBD 9.
10. From your FBD you can say that $\quad F_{\mathrm{b}}=$ $\qquad$
11. We want to know the density of the unknown material, $\rho_{\mathrm{u}}$. If we could find its weight we could use $W_{\mathrm{u}}=\rho_{\mathrm{u}} g V_{\mathrm{u}}$ since we know $g$ and we can measure $V_{\mathrm{u}}$ using our overflow system.
So to find $\rho_{\mathrm{u}}$ we just need $W_{\mathrm{u}}$. We know $W_{\mathrm{c}}$, so if we knew $F_{\mathrm{b}}$ we could find $\rho_{\mathrm{u}}$ using Equation 6.
From equations 4 and 5 you know two different ways of finding $F_{\mathrm{b}}$. So you're ready to go.
So we have two equations, $F_{\mathrm{b}}=W_{\mathrm{c}}+W_{\mathrm{u}}$, and $W_{\mathrm{u}}=\rho_{\mathrm{u}} g \mathrm{~V}_{\mathrm{u}}$
Combine the two equations and solve for $\rho_{\mathrm{u}}$. Show your algebra below.

Take the data required below to determine the density of the unknown material. You'll need to determine the total buoyant force, $F_{\mathrm{b}}$ by either of the methods suggested in Part I. Record $F_{\mathrm{b}}$ and $\rho_{\mathrm{u}}$ in the table.
12. $W_{\text {cork }}=$ $\qquad$ N (from part II)
13. $F_{\mathrm{b}}=$ $\qquad$ N
14. $V_{\mathrm{unknown}}=$ $\qquad$ ml
15. $V_{\mathrm{unknown}}=$ $\qquad$ $\mathrm{m}^{3}$
16. $\rho_{\mathrm{u}}=$ $\qquad$ $\mathrm{kg} / \mathrm{m}^{3}$
17. The unknown material is concrete. Check on line to see if your result is reasonable. Note that concrete varies in density depending on its manufacture.

Show your calculations of the buoyant force, $F_{\mathrm{b}}$ and the unknown density, $\rho_{\mathrm{u}}$.

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